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Integrated Models for Shipping a Vendor's Final Production Batch to a Single Buyer Under Linearly Decreasing Demand for Consignment Policy

(Model Integrasi untuk Penghantaran Bac Pengeluaran Terakhir kepada Pembeli di bawah Permintaan Menyusut Secara Linear bagi Polisi Konsainmen)

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ABSTRACT

This paper considers the problem of a vendor or manufacturer supplying a final production batch to a single buyer under linearly decreasing demand for a finite time horizon. The vendor manufactures the product at a finite rate and ships the output to the buyer. In this model we considered the case where the holding cost at the vendor is greater than the buyer and propose a consignment model. The objective was to minimize the total cost of stock transfer from vendor to buyer and stock holding at the vendor and the buyer. We derived the structure of the optimal solution and illustrate the proposed models with numerical examples.

Keywords: Consignment policy; integrated; single buyer; single vendor; varied demand

ABSTRAK

Kertas ini membincangkan masalah penjual atau pengeluar membekalkan bahagian pengeluaran terakhir kepada seorang pembeli di bawah permintaan menyusut secara linear dengan selang masa terhingga. Penjual mengeluarkan produk pada kadar terhingga dan menghantar output kepada pembeli. Dalam model ini kami membincangkan kes dengan kos pegangan penjual adalah lebih tinggi daripada pembeli dan mencadangkan model konsainmen. Objektifnya adalah untuk meminimumkan jumlah kos penghantaran stok daripada penjual kepada pembeli dan kos pemegangan stok bagi penjual dan pembeli. Kami membina struktur penyelesaian optimum dan menggambarkan model yang dicadangkan melalui contoh-contoh berangka.

Kata kunci: Integrasi; pembeli tunggal; penjual tunggal; permintaan berubah; polisi konsainmen

INTRODUCTION

In classical economic order quantity (EOQ) models, the replenishment quantity for a single product is determine by minimizing the total cost per unit time. This total cost include the inventory ordering and holding cost. In this model, the demand rate is assumed to be constant and known over an infinite planning horizon. Following this model, the vendor's and buyer's inventory problems are treated in isolation. The EOQ formula can give an optimal solution for them respectively. However this independent decision behavior cannot assure that the two parties as a whole reach the optimal state. Therefore, due to this need, the concept of integrated production-inventory problem is introduced to refine the EOQ model.

Integrated production-inventory problem with a single-vendor single-buyer with a constant demand rate has been studied extensively by a number of researchers. Goyal (1977) is probably one of the first to investigate this problem and initiated the idea of the integrated model where the objective is to minimize the total relevant costs for both the vendor and the buyer. This model is suitable when a collaborative arrangement between the buyer and the vendor is enforced by some contractual agreement.

Banerjee (1986) considered the vendor manufacturing the stock at a finite rate and delivering the whole batch to the buyer as a single shipment -a 'lot for lot' model. Goyal (1988) demonstrated how lower cost policies could be obtained by allowing a production batch to be split and delivered as a number of shipments. Lu (1995) set out the optimal production and shipment policy when the shipments sizes are all equal. Goyal (1995) also demonstrated how lower cost policies are sometimes obtained when successive shipment sizes are increased by a ratio which is equal to production rate divided by the demand rate. Hill (1999) derived the form of the optimal policy if shipment sizes vary. This consists of a number of shipments which is increased using the ratio in Goyal (1995) followed by a number of equal-sized shipments.

A common assumption in these models is the unit holding cost is more expensive for the buyer than for the vendor. However, studies on consignment stock policies by Braglia and Zavanella (2003) and Valentini and Zavanella (2003) suggest that there are some situations that exist in the industry where the reverse is true. According to Hill and Omar (2006), the underlying principle of the consignment stock is that the vendor takes responsibility for managing stock. In order to minimise the stock held by the vendor, the vendor will ship all the available stocks whenever a delivery is ready for shipment.

In all the above models, the demand rate is constant over an infinite time horizon. Recently, Omar (2009) had discussed the integrated policy for shipping a vendor's final production to a buyer with a linearly decreasing demand rate over a finite time horizon. In this paper, we introduced a consignment model with equal and different shipment sizes and we illustrated the effectiveness of the model by numerical examples. We end with the conclusions.

Mathematical Formulation

Assumptions and Notation

The following assumptions are made in this model:

1. The operating environment is deterministic where the demand rate for the finished product at time t is $f(t) = a(1-\frac{t}{H})$ for $t \in (0,H)$. a is the initial demand rate with $a > 0$ and H is the time horizon.
2. At time zero the buyer holds a quantity x in stock (no shortages at the buyer are allowed). In Omar's model (2009), he assumed that the initial stock, x is given. However, in this model x is determined by the quantity of the first shipment which is delivered immediately after the buyer's inventory reach zero level at time t_1, where x is the amount of stock which available from time 0 to t_1 and it follows that $x = \int_0^{t_1} f(t)dt$.
3. The finite production rate is P units per unit time and $P > a$.
4. The production set up cost is irrelevant since there is the final batch.
5. Shipment's operations are carried out during production uptime and downtime until all demand is satisfied.

The following notational scheme is used throughout this model:

1. There is a fixed ordering or shipment cost of A_2.
2. There is an inventory carrying cost for the vendor of h_1 per unit per unit time for finished product.
3. There is an inventory carrying cost for the buyer of h_2 per unit per unit time.
4. n is the number of shipments.
5. q_i is the size of the ith shipment.
6. x is the initial stock held at the buyer when the final production is about to start.
7. $C(n)$ is the total cost for the system.

Figure 1 illustrates the inventory level at the vendor, buyer and the system with 4 shipments. This batch is divided into two successive periods, that is the production uptime $(0 \le t \le t_p)$ with the stock level at time t equals to $y_2(t)=Pt+x-\int_0^t f(t)dt$, and downtime $(t_p \le t \le H)$ with the stock level equals to $y_1(t)=\int_t^H f(t)dt$.

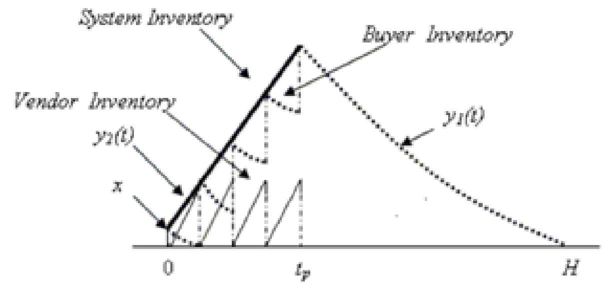

FIGURE 1. Inventory level for the case $h_1 > H_2$

The production starts at $t_0 = 0$ at rate P where $P > a$. The initial stock at the buyer which is x units will finish up at time t_1, where the first shipment, q_1 $(=Pt_1)$, is replenished. $q_2, q_3, \ldots, q_n$ are the shipment quantity for the subsequent shipments, and they could be equal or vary. The optimal policy for this case may lead to the suppression of the vendor's inventory and using the buyer's warehouse to stock finished product. Therefore, we wish to minimise the stock held by the vendor and so the vendor ships all the stock available whenever a shipment is made.
The total cost function, $C(n)$, is:

$$C(n) = n(A_2) + h_2 TSS + (h_1 - h_2)TVS. \quad (1)$$

Structurally the cost function is identical to the Omar's (2009) model. The constants h_1 and h_2 are interchanged and the last term on the right hand side is now multiplied by the total time-weighted vendor stock, TVS, which is given by:

$$TVS = \frac{1}{2}\sum_{i=1}^{n} q_i\left(t_i - t_{i-1}\right). \quad (2)$$

The total time-weighted system stock, TSS, is the area under the curve as shown in Figure 1 which can be written as follow:

$$TSS = \int_{t_o}^{t_p} y_2\, dt + \int_{t_p}^{H} y_1\, dt$$
$$= \frac{1}{6}aH^2 - \frac{1}{2P}\left(\frac{1}{2}aH - x\right)^2. \quad (3)$$

For any integer n and for any given policy, for example equal-lots policy (see Omar (2009)), we have to determine the values of $t_1, t_2, \ldots, t_n$. We shall increase or decrease the value of n until we find the first minimum of $C(n)$.

Note that the first shipment size, q_1 is equal to Pt_1. Since stockout is not allowed, the time to produce ith shipment must be less or equal to the time for the buyer to finish up the previous (i-1)th shipment. It follows that, *where:*

$$P(t_2 - t_1) \geq q_2 \ or \ t_2 \geq t_1 + \frac{q_2}{P} \ = \frac{q_1}{P} + \frac{q_2}{P},$$

$$P(t_3 - t_2) \geq q_3 \ or \ t_3 \geq t_2 + \frac{q_3}{P} \ = \frac{q_1}{P} + \frac{q_2}{P} + \frac{q_3}{P},$$

$$\vdots$$

$$P(t_i - t_{i-1}) \geq q_i \ or \ t_i \geq t_{i-1} + \frac{q_i}{P} = \frac{q_1}{P} + \frac{q_2}{P} + \frac{q_3}{P} + \ldots + \frac{qn}{P}$$

$$= \frac{1}{P}\sum_{j=1}^{i} q_j \ldots, n.$$

where:

$$\int_{t_{i-1}}^{t_i} f(t)\,dt = q_{i-1}.$$

Solving for t_i, we get:

$$t_i = H\left\{1 - \sqrt{1 - \frac{2}{aH}\left[a\left(\frac{q_{i-1}}{P}\right) - \frac{a}{2H}\left(\frac{q_{i-1}}{P}\right)^2 + q_{i-1}\right]}\right\}.$$

From these arguments, we can establish the following constraint optimization problem,

Minimum $\quad C(n) \quad$ (4)

Subject to:

$$H\left\{1 - \sqrt{1 - \frac{2}{aH}\left[a\left(\frac{qi-1}{P}\right) - \frac{a}{2H}\left(\frac{qi-1}{P}\right)^2 + qi - 1\right]}\right\} \quad (5)$$

and

$$\sum_{j=1}^{n} q_i = D - x_i \quad (6)$$

The shipment time for this policy is:

$$t_i = \frac{q_i}{P} + \frac{q_{i-1}}{P} \quad i = 2, 3, \ldots, n. \quad (7)$$

The computer algorithm for the solution procedure is:

1. Let n=1,
2. Set t_0=0, t_{n+1}=H,
3. Determine q_i i=1, 2, …, n which satisfied constraints *(5)* and *(6)* if exist,
4. Compute t_i, i=1, 2, …, n using *(7)* and *C(n)* using *(1)*,
5. Set *C(n)* as $C(n^*)$. Increase n by 1 and repeat step 3 to 4. Stop when $C(n) \geq C(n^*)$.

The basic idea of the above algorithm is to start with n = 1. Next, we increase n to improve the total system cost until the first n = n^* that satisfies the conditions $C(n^*) < C(n^*-1)$ and $C(n^*) < C(n^*+1)$.

Note that if we consider the equal shipments size, we have to change constraint (5) to the following constraint:

$$q_{i,j} = q_{i,j+1}. \quad (8)$$

NUMERICAL EXAMPLES

To show the effectiveness of the proposed policies we adopt the same numerical examples as Omar (2009). For easy reference, the parameter values are restated here;

A_2=25, a=200, b=40, D=500, H=5, h_1=7, h_2=5, P=1000.

Table 1 gives the minimum total cost for the different shipments size policy for different values of n. This table shows how an increase in the number of shipments can affect the total cost of the system. From the above algorithm, the optimum total cost, $C(n^*)$. is 3728.34 when n^*= 4 where the sizes of each shipments are 24.40, 148.54, 161.10 and 161.10 respectively with shipment times at 0.024, 0.173, 0.334 and 0.495.

TABLE 1: Cost for the different shipments size policy

n	Total cost	q_1	q_2	q_3	q_4	q_5
1	3927.57	419.60	-	-	-	-
2	3788.54	151.94	318.14	-	-	-
3	3740.13	37.03	227.18	228.42	-	-
4	3728.34	24.40	148.54	161.10	161.10	-
5	3733.29	17.42	105.60	124.49	124.49	124.49

TABLE 2. Cost for the equal shipments size policy

n	Total cost	q_1	q_2	q_3	q_4	q_5
1	3927.57	419.60	-	-	-	-
2	3801.73	227.74	227.74	-	-	-
3	3764.66	156.40	156.40	156.40	-	-
4	3755.88	119.12	119.12	119.12	119.12	-
5	3759.65	96.19	96.19	96.19	96.19	96.19

Similarly, Table 2 shows the minimum total cost for the equal shipment size policy for different values of n. The optimum total cost, $C(n^*)$ is 3755.88 when $n^*= 4$ and its shipments size 119.12. As expected, the different shipments size policy is always better than equal shipments size policy. For our problem, the optimum total cost from the equal shipments size policy is 0.74% higher than the different shipments size.

Table 3 gives the optimum total cost for variation of P. When the production rate increases, the stock become immediately available, so the number of shipments will decreases and the optimum total cost will be increased.

TABLE 3: Different P – different shipments size

P	n	Total cost	q_1	q_2	q_3	q_4	q_5
1000	4	3728.34	24.40	148.54	161.10	161.10	-
2000	3	3984.50	95.57	197.46	197.46	-	-
3000	2	4059.86	212.06	273.91	-	-	-
4000	2	4097.38	220.91	268.11	-	-	-
5000	2	4120.39	226.41	264.58	-	-	-

CONCLUSION

This paper proposes a mathematical model for the vendor's final production shipment to a single buyer when the demand rate is linearly decreasing over a finite time horizon. We considered two policies where shipments size are different and equal. As expected the policy with different shipments size is superior than the equal shipments size. Furthermore, this model can be easily extended to *n*-batches model.

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